Light-Cone HQET Sum Rules for the $B \to \pi$ Transition with $1/m_Q$ Corrections

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Abstract

The $B \to \pi \ell \nu$ weak decay form factors are calculated via light-cone sum rules within the framework of the heavy quark effective theory. We calculate the leading and the relevant sub-leading universal form factors. Our results are matched to the known soft pion limit. We also address the large pion energy limit of our sum rule results. Our results are compared with that of other approaches.

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I. INTRODUCTION

It is well-known that the weak $B \to \pi$ transition is important for the extraction of the Cabbibo-Kobayashi-Maskawa matrix element V_{ub} from semileptonic decays, and for the measurement of CP violation from non-leptonic decays. In order to achieve these goals advances have to be made both on the experimental and theoretical sides. On the theoretical side an important task is to reduce the uncertainties in the calculations of the relevant hadronic matrix element, represented by

$$\langle \pi(p) | \, \bar{u} \, \gamma^\mu \, b \, | B(P) \rangle \; = \; f_+(q^2) \left[(P+p)^\mu - \frac{m_B^2 - m_\pi^2}{q^2} \, q^\mu \right] + f_0(q^2) \, \frac{m_B^2 - m_\pi^2}{q^2} \, q^\mu \,, \quad (1)$$

where $q = P - p$ is the momentum transfer to the leptons. Up to now, the form factors have mostly been calculated via light-cone (LC) sum rules [1] in full QCD. Instead, here we will work in the context of the heavy quark effective theory (HQET) [2]. The reasons are as follows:

(i) Unlike for heavy-to-heavy transitions an application of HQET methods does not significantly simplify the analysis for heavy-to-light transitions. Nevertheless, the systematic nature of HQET allows one to identify and estimate uncertainties in the heavy-to-light transitions more easily.

(ii) For an analysis involving the B meson HQET is the correct approximation method of QCD. In certain parts of the phase space of the semileptonic decay, the pion is not very energetic so that HQET may still be valid.

(iii) The $B \to \pi\pi$ amplitude has been shown to be factorizable at the leading order of the $1/m_b$ expansion [3]. Knowledge of the form factors based on HQET is needed so as to consistently apply the factorization approach.

(iv) Furthermore, it is still controversial whether the time-like transition $B \to \pi$ at large recoil is governed by perturbative or non-perturbative QCD [3, 4]. If non-perturbative effects dominate, then the use of HQET for the $B \to \pi$ transitions can be fully justified.

(v) In addition, the results of HQET can also be applied to $D \to \pi(K)$ transitions. In HQET, a model-independent analysis of $B(D) \to \pi$ transitions to order $1/m_Q$ has been done in Ref. [5, 6] and for $B(D) \to \rho$ in Ref. [7].

HQET simplifies the analysis by introducing a set of universal functions. However, in order to obtain information on the universal functions themselves, some nonperturbative techniques, such as light-cone sum rules or lattice simulations, must be used in addition. Here we adopt the LC sum rule method [8, 9] which is suitable for the calculation of form factors when light energetic hadrons are involved. To our knowledge, the LC-HQET sum rules were first applied in Ref. [10]. In Ref. [11], they were used at the leading order of HQET for $B \to \pi$ transitions. However, the results in this paper differ from those in [11].

Note that we distinguish between LC-HQET sum rules and LC-QCD sum rules with the $1/m_Q$ expansion [12]. The main reason is that the ways to include radiative corrections are different. Another reason is that for $H \to \pi$ transitions, there is a subtle difference between the two types of sum rules, as will be discussed in the paper.

In this paper we apply LC-HQET sum rules to the $H \to \pi$ transition to order $1/m_Q$. In the next section we give a brief review on the application of HQET to the decay $B \to \pi\ell\nu$. In section III, the leading and next-to-leading order universal functions are calculated by using LC-HQET sum rules. In Section IV we present our numerical results. Section V contains a summary and discussion, and a comparison with other approaches.

II. THE HEAVY QUARK EXPANSION

In HQET, the velocity of heavy quark Q, v, is a well defined quantity. The heavy quark field can be represented by the velocity-dependent field,

$$h_v(x) = \exp(im_Q \, v{\cdot}x) \, P_+ \, Q(x) \,, \tag{2}$$

where $P_+ = \dfrac{1 + \not{v}}{2}$ projects onto the upper component of the heavy quark field $Q(x)$. To the order $1/m_Q$, the effective Lagrangian is given by [13]

$$\mathcal{L}_{\rm HQET} = \bar{h}_v \, iv{\cdot}D \, h_v + \frac{1}{2m_Q} \left[O_{\rm kin} + C_{\rm mag}(\mu) \, O_{\rm mag} \right] \,, \tag{3}$$

where the gauge-covariant derivative $D_\mu = \partial_\mu - ig_s T^a A^a_\mu$ generates the residual momentum k_μ, and

$$O_{\rm kin} = \bar{h}_v \, (iD)^2 h_v \,, \qquad O_{\rm mag} = \frac{g_s}{2} \, \bar{h}_v \, \sigma_{\alpha\beta} G^{\alpha\beta} h_v \,. \tag{4}$$

$O_{\rm kin}$ describes the kinetic energy of the heavy quark in the hadron, and $O_{\rm mag}$ the heavy quark chromomagnetic energy. The equation of motion, $i\,v \cdot D\,h_v = 0$, is exactly satisfied. The higher dimension operators are treated as perturbative power corrections. In the leading logarithmic approximation, the renormalization factor $C_{\rm mag}(\mu)$ is given by

$$C_{\rm mag}(\mu) = r(\mu)^{-3} \,, \qquad r(\mu) = \left[\frac{\alpha(\mu)}{\alpha(m_Q)} \right]^{\frac{3}{33-2n_f}} \,, \tag{5}$$

where n_f is the number of quarks lighter than the heavy quark Q.

In the framework of HQET, it is convenient to work in the matrix representation for the description of the hadrons, in which wave functions of heavy hadron are only dependent on the heavy quark symmetry and their Lorentz transformation properties. The ground-state pseudo-scalar and vector heavy mesons are described by the so-called spin wave function

$$\mathcal{M}(v) = P_+ \begin{cases} -\gamma_5 \,, & \text{for } J^P = 0^- ; \\ \not{\epsilon} \,, & \text{for } J^P = 1^- \end{cases} \tag{6}$$

with ϵ^μ being the polarization vector. The form factors are considered as functions of the kinematic variable

$$v \cdot p = \frac{m_H^2 + m_\pi^2 - q^2}{2m_H} \,. \tag{7}$$

Using the mass-independent normalization of the heavy meson state $|H(v)\rangle = m_H^{-1/2}|H(P)\rangle$, the form factors can be re-defined as

$$\langle \pi(p)| \, \bar{u} \, \gamma_\mu \, Q \, |H(v)\rangle = 2 \left[f_1(v \cdot p) \, v_\mu + f_2(v \cdot p) \, \widehat{p}_\mu \right] \,, \tag{8}$$

where the dimensionless variable is $\widehat{p}^\mu = \dfrac{p^\mu}{v \cdot p}$. The relation between the form factors in Eqs. (1) and (8) is given by [6]

$$f_+(q^2) = \sqrt{m_H} \left\{ \frac{f_2(v \cdot p)}{v \cdot p} + \frac{f_1(v \cdot p)}{m_H} \right\} \,, \tag{9}$$

$$f_0(q^2) = \frac{2}{\sqrt{m_H}} \frac{m_H^2}{m_H^2 - m_\pi^2} \left\{ \left[f_1(v \cdot p) + f_2(v \cdot p) \right] - \frac{v \cdot p}{m_H} \left[f_1(v \cdot p) + \widehat{p}^2 \, f_2(v \cdot p) \right] \right\} \,. \tag{10}$$

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The form factors can be systematically analyzed order by order in terms of powers of $1/m_Q$. Let us begin with the expansion of the heavy-light vector current,

$$\langle \pi(p)|\bar{q}\,\gamma^\mu\,Q|H(v)\rangle = \sum_i C_i(\mu)\,\langle \pi(p)|J_i|H(v)\rangle$$
$$+ \sum_j \frac{1}{2m_Q}B_j(\mu)\langle \pi(p)|O_j|H(v)\rangle + \mathcal{O}(\frac{1}{m_Q^2})\,. \qquad (11)$$

In the limit of massless light quarks, a convenient basis of the above operators is [14]

$$\begin{aligned}
J_1 &= \bar{q}\,\gamma^\mu h_v\,, & J_2 &= \bar{q}\,v^\mu h_v\,,\\
O_1 &= \bar{q}\,\gamma^\mu\,i\slashed{D}\,h_v\,, & O_4 &= \bar{q}\,(-iv\cdot\overleftarrow{D})\,\gamma^\mu h_v\,,\\
O_2 &= \bar{q}\,v^\mu\,i\slashed{D}\,h_v\,, & O_5 &= \bar{q}\,(-iv\cdot\overleftarrow{D})\,v^\mu h_v\,,\\
O_3 &= \bar{q}\,iD^\mu h_v\,, & O_6 &= \bar{q}\,(-i\overleftarrow{D^\mu})\,h_v\,.
\end{aligned} \qquad (12)$$

The corresponding Wilson coefficients are given by [14, 15]

$$B_1(\mu) = C_1(\mu) = r^2\,, \qquad B_2(\mu) = \frac{1}{2}B_3(\mu) = C_2(\mu) = 0\,,$$
$$B_4(\mu) = \frac{34}{27}r^2 - \frac{4}{27}r^{-1} - \frac{10}{9} + \frac{16}{3}r^2\,\ln r\,, \qquad (13)$$
$$B_5(\mu) = -\frac{28}{27}r^2 + \frac{88}{27}r^{-1} - \frac{20}{9}\,, \qquad B_6(\mu) = -2\,r^2 - \frac{4}{3}r^{-1} + \frac{10}{3}\,.$$

At the leading order of the $1/m_Q$ expansion, the matrix element of the relevant current $\bar{q}\,\Gamma\,h_v$ can be written as

$$\langle \pi(p)|\,\bar{q}\,\Gamma\,h_v\,|H(v)\rangle = -\text{Tr}\Big\{ \gamma_5\Big[L_a(v\cdot p,\,\mu) + \slashed{p}\,L_b(v\cdot p,\,\mu)\Big]\Gamma\,\mathcal{M}(v) \Big\}\,, \qquad (14)$$

where the universal functions $L_\alpha(v\cdot p,\mu)$ $(\alpha = a,\,b)$ depend on the kinematic variable $v\cdot p$, but not on the heavy quark mass m_Q.

At the next-to-leading order in the heavy-quark expansion, the $1/m_Q$ corrections coming from both the effective current and the effective Lagrangian of HQET will appear as follows. Matrix elements of the operators O_1, O_2, and O_3 in the effective current can be expressed by the generic structure

$$\langle \pi(p)|\,\bar{q}\,(\Gamma\,iD)_\mu h_v\,|H(v)\rangle$$
$$= -\text{Tr}\Big\{ \Big[(F_1\,v_\mu + F_2\,\widehat{p}_\mu + F_3\,\gamma_\mu)\,\gamma_5 + (F_4\,v_\mu + F_5\,\widehat{p}_\mu + F_6\,\gamma_\mu)\,\gamma_5\,\slashed{\widehat{p}}\Big]\Gamma\,\mathcal{M}(v) \Big\}\,, \quad (15)$$

where the universal functions $F_i(v\cdot p,\mu)$ $(i = 1,\cdots,6)$ are also m_Q-independent. Matrix elements of the operators O_4, O_5, and O_6 are not independent, and can be obtained from the above structures Eqs.(14) and (15). Additionally, corrections coming from insertions of the operators O_{kin} and O_{mag} into matrix elements of the leading-order currents can be described by six additional universal functions $K_\alpha(v\cdot p,\mu)$ $(\alpha = a,\,b)$ and $S_i(v\cdot p,\mu)$ $(i = 1,\cdots,4)$, which are defined by matrix elements of the time-ordered products

$$\langle \pi(p)|\,i\int d^4y\,T\Big\{\bar{q}\,\Gamma\,h_v(0),\,O_{\text{kin}}(y)\Big\}|H(v)\rangle = -\text{Tr}\Big\{ \gamma_5\,(\,K_a + \slashed{\widehat{p}}\,K_b\,)\,\Gamma\,\mathcal{M}(v) \Big\}\,, \quad (16)$$

$$\langle \pi(p)|\,i\int d^4y\,T\Big\{\bar{q}\,\Gamma\,h_v(0),\,O_{\text{mag}}(y)\Big\}|H(v)\rangle$$
$$= -\text{Tr}\Big\{ \Big[(iS_1\,\widehat{p}_\alpha\gamma_\beta + S_2\,\sigma_{\alpha\beta})\,\gamma_5 + (iS_3\,\widehat{p}_\alpha\gamma_\beta + S_4\,\sigma_{\alpha\beta})\,\gamma_5\,\slashed{\widehat{p}}\Big]\Gamma\,P_+\,\sigma^{\alpha\beta}\,\mathcal{M}(v) \Big\}\,. \quad (17)$$

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The form factors in Eq. (8) can then be expanded as follows after doing the appropriate traces [6],

$$f_1 = C_1 L_a + \frac{1}{2m_Q}\left\{ C_1\left[\mathcal{F}_a^1 + (K_a + C_{\text{mag}}\,\mathcal{S}_a)\right] - B_4\,(\bar{\Lambda} - v\cdot p)\,L_a\; -\; B_5\,(\bar{\Lambda} - v\cdot p)\right.$$
$$\left. \times (L_a + L_b)\; +\; B_6\left[\mathcal{F}_a^3 - \bar{\Lambda}(L_a + L_b)\right]\right\} + \mathcal{O}(\frac{1}{m_Q^2})\,, \qquad (18)$$

$$f_2 = C_1 L_b + \frac{1}{2m_Q}\left\{ C_1\left[\mathcal{F}_b^1 + (K_b + C_{\text{mag}}\,\mathcal{S}_b)\right] - B_4\,(\bar{\Lambda} - v\cdot p)\,L_b\right.$$
$$\left. + B_6\left[\mathcal{F}_b^3 + v\cdot p\,(L_a + L_b)\right]\right\} + \mathcal{O}(\frac{1}{m_Q^2})\,, \qquad (19)$$

where $\mathcal{F}_\alpha^i$ $(i = 1,\,3)$ and $\mathcal{S}_\alpha$ are defined by

$$\begin{aligned}
\mathcal{F}_a^1 &= F_1 + 2F_3 - \hat{p}^2 F_5, & \mathcal{F}_a^3 &= F_1 - F_3 + F_4\,,\\
\mathcal{F}_b^1 &= F_2 + F_4 + 2F_5 - 4F_6, & \mathcal{F}_b^3 &= F_2 + F_5 - F_6\,,\\
\mathcal{S}_a &= -2S_1 + 6S_2 + 2\hat{p}^2 S_3\,, & \mathcal{S}_b &= 2S_1 - 2S_3 + 6S_4\,.
\end{aligned} \qquad (20)$$

It should be mentioned that consequences of the heavy quark symmetry and the equations of motion for the heavy and light quark fields make only two (F_5 and F_6) of the six form factors F_i's independent [6]. This means that one can re-express $\mathcal{F}_\alpha^i$ by L_a, L_b and F_6 as follows

$$\begin{aligned}
\mathcal{F}_a^1 &= -(\bar{\Lambda} - 2v\cdot p)L_a + v\cdot p\,\hat{p}^2 L_b + 4F_6\,, & \mathcal{F}_a^3 &= v\cdot p\,L_a + \bar{\Lambda}L_b + 2F_6\,,\\
\mathcal{F}_b^1 &= -v\cdot p\,L_a - \bar{\Lambda}L_b - 4F_6\,, & \mathcal{F}_b^3 &= -v\cdot p\,L_a - \bar{\Lambda}L_b - 2F_6\,.
\end{aligned} \qquad (21)$$

Note that between the two independent universal functions from the effective current corrections only $F_6(v\cdot p, \mu)$ is relevant to the $H \to \pi$ matrix element in Eq. (1)

$$F_6 = -\frac{1}{2}\left(\mathcal{F}_b^1 + \mathcal{F}_a^3\right). \qquad (22)$$

III. THE LIGHT-CONE HQET SUM RULES

Our aim is to calculate the independent form factors given in the last section by LC-HQET sum rules. In subsection A, the leading universal functions L_α ($\alpha = a,\,b$) are calculated. The relevant sub-leading universal functions, F_6 and $K_\alpha + C_{\text{mag}}\mathcal{S}_\alpha$, will be calculated in subsections B and C, respectively.

A. Leading order

Let us begin with the following 2-point vacuum-pion correlation function as depicted in Fig. 1:

$$\begin{aligned}
F_\mu(\lambda, \omega) &= i\int \mathrm{d}^4 x\, e^{ik\cdot x}\,\langle \pi(p)|\,T\!\left\{\bar{u}(x)\gamma_\mu h_v(x),\, \bar{h}_v(0)i\gamma_5 d(0)\right\}|0\rangle\\
&= F_a(\lambda, \omega)\,v_\mu\, +\, F_b(\lambda, \omega)\,\hat{p}_\mu\,,
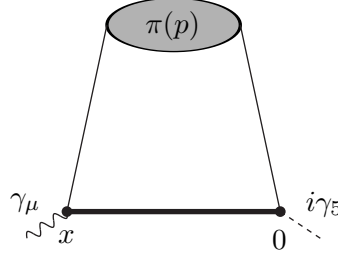\end{aligned} \qquad (23)$$

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FIG. 1: *The diagrammatic representation of the 2-point vacuum-pion correlation function Eq. (23), where the heavy solid line represents the heavy quark.*

where $\lambda = 2v \cdot p$ and $\omega = 2v \cdot k$. On the one hand, by inserting a complete set of intermediate states with the same quantum numbers as the H meson between the current in the above vacuum-pion correlation function, we obtain the hadronic representation

$$F_\alpha^{\text{Hadr}}(\lambda, \tilde{\omega}) \;=\; 2F \frac{L_\alpha(\lambda)}{2\bar{\Lambda} - \tilde{\omega}} + resonances\,, \tag{24}$$

where $\tilde{\omega} = 2v \cdot \tilde{k}$ with $\tilde{k}$ being $k + p$. $\bar{\Lambda}$ is the heavy meson mass as defined in the HQET. *Note that it is $\tilde{\omega}$ that is the relevant energy of the quark system.* The decay constant F is defined as

$$\langle H(v)|\bar{h}_v \Gamma d|0\rangle \;=\; \frac{i}{2} F(\mu)\text{Tr}\Big[\Gamma \mathcal{M}(v)\Big]\,, \tag{25}$$

with Γ being an arbitrary Dirac matrix.

On the other hand the correlation function can be calculated by expanding the T-product of the currents near the light-cone at $x^2 = 0$. In our calculations, the chiral limit $p^2 = m_\pi^2 = 0$ is taken. By adopting the fixed-point gauge in which

$$x_\mu A^\mu(x) = 0\,, \tag{26}$$

the following complete heavy quark propagator equals the free one,

$$\langle 0|T\{\, h_v(x)\, \bar{h}_v(0)\, \}|0\rangle \;=\; \int_0^\infty \mathrm{d}\,t\, \delta(x - vt) \frac{1 + \not{v}}{2}\,. \tag{27}$$

The matrix elements of the nonlocal vacuum-to-pion operators can be parameterized by pion wave functions on the light-cone. The LC wave functions are classified according to their twist and the number of partons they describe. Here, let $\mathcal{WF}(n)$ denote the n-particle light-cone wave function. One can easily find that at the leading order of HQET only the $\mathcal{WF}(2)$s are relevant. Up to twist four, the matrix elements are parameterized as follows [8, 16]:

$$\langle \pi(p)|\bar{u}(x)\gamma_\mu \gamma_5 d(0)|0\rangle \;=\; -i f_\pi p_\mu \int_0^1 \mathrm{d}u\, e^{iu\,px}\Big(\varphi_\pi(u) + x^2 g_1(u)\Big)$$

$$+ f_\pi \Big(x_\mu - \frac{x^2 p_\mu}{px}\Big) \int_0^1 \mathrm{d}u\, e^{iu\,px} g_2(u)\,,$$

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$$\langle \pi(p)|\bar{u}(x)i\gamma_5 d(0)|0\rangle = f_\pi \mu_\pi \int_0^1 \mathrm{d}u \; e^{iu\,px}\varphi_p(u)\,,$$

$$\langle \pi(p)|\bar{u}(x)\sigma_{\mu\nu}\gamma_5 d(0)|0\rangle = i\frac{f_\pi \mu_\pi}{6}\left(p_\mu x_\nu - p_\nu x_\mu\right)\int_0^1 \mathrm{d}u \; e^{iu\,px}\varphi_\sigma(u)\,, \qquad (28)$$

with $\mu_\pi = m_\pi^2/(m_u+m_d)$. Here, φ_π is the leading twist-2 wave function, g_1 and g_2 are twist-4 wave functions and $\varphi_p, \varphi_\sigma$ twist-3 wave functions. In the fixed-point gauge, the operator $\Pi_G = P\exp\left\{ig_s\int_0^1 d\alpha\; x_\mu A^\mu(\alpha x)\right\}$ is unity. Considering the identity

$$\gamma_\mu \gamma_\nu = -i\sigma_{\mu\nu} + g_{\mu\nu}\,, \qquad (29)$$

one can obtain $F_\alpha^{\mathrm{HQET}}(\lambda,\tilde{\omega})$ in terms of the light-cone wave functions:

$$F_a^{\mathrm{LO}}(\lambda,\tilde{\omega}) = \frac{f_\pi}{2}\int_0^1 \mathrm{d}u \int_0^\infty i\,\mathrm{d}t\, e^{i\frac{\tilde{\omega}}{2}t}\left[\mu_\pi \varphi_p(u) - \left(\frac{\mu_\pi}{6}\varphi_\sigma(u) - \frac{2}{\lambda}g_2(u)\right)\frac{\mathrm{d}}{\mathrm{d}u}\right]e^{-i\bar{u}\frac{\lambda}{2}t}\,, \qquad (30)$$

$$F_b^{\mathrm{LO}}(\lambda,\tilde{\omega}) = \frac{f_\pi}{2}\int_0^1 \mathrm{d}u \int_0^\infty i\,\mathrm{d}t\, e^{i\frac{\tilde{\omega}}{2}t}\left[\frac{\lambda}{2}\varphi_\pi(u) + \left(\frac{\mu_\pi}{6}\varphi_\sigma(u) - \frac{2}{\lambda}g_2(u)\right)\frac{\mathrm{d}}{\mathrm{d}u}\right.$$
$$\left. -\frac{2}{\lambda}g_1(u)\frac{\mathrm{d}^2}{\mathrm{d}u^2}\right]e^{-i\bar{u}\frac{\lambda}{2}t}\,, \qquad (31)$$

where here and below $\bar{u} = 1 - u$, and the superscript LO denotes the leading order in the $1/m_Q$ expansion.

Setting F_α^{Hadr} equal F_α^{HQET} defines the LC-HQET sum rules. The quark-hadron duality assumption is used to substitute the unknown *resonances* by the HQET result after a dispersion integration above some given threshold ω_c,

$$resonances = \frac{1}{\pi}\int_{\omega_c}^\infty \mathrm{d}\nu \frac{\mathrm{Im}F_\alpha^{\mathrm{HQET}}(\lambda,\nu)}{\nu - \tilde{\omega}} + subtraction\,. \qquad (32)$$

The HQET spectral density can be obtained by the following double Borel transformation,

$$\frac{1}{\pi}\mathrm{Im}F_\alpha^{\mathrm{HQET}}(\lambda,\nu) = \widehat{B}_{\frac{1}{\nu}}^{(-\frac{1}{\tau})}\widehat{B}_\tau^{(\tilde{\omega})}F_\alpha^{\mathrm{HQET}}(\lambda,\tilde{\omega}) \qquad (33)$$

with the Borel transformation being defined as

$$\widehat{B}_Y^{(X)} = \lim_{\substack{X\to\infty,\; Y=\frac{-X}{n}\,\mathrm{fixed}\\ n\to\infty,}} Y\frac{(-X)^n}{\Gamma(n)}\frac{\mathrm{d}^n}{\mathrm{d}X^n} \qquad (34)$$

whose property, $B_\tau^{(\tilde{\omega})}e^{\rho\tilde{\omega}} = \delta(\rho - 1/\tau)$, is very useful in the calculation here. Evidently, before doing Borel transformations, one should first perform the Wick rotation on t.

Finally, by performing a Borel transformation $\widehat{B}_T^{(\tilde{\omega})}$ on both sides of the sum rule, so as to enhance the ground state contribution, suppress higher twist terms and remove the *subtraction*, one can obtain the sum rules,

$$L_\alpha(\lambda) = \frac{1}{2F}e^{2\bar{\Lambda}/T}\int_0^{\omega_c} \mathrm{d}\nu \frac{1}{\pi}\mathrm{Im}F_\alpha^{\mathrm{LO}}(\lambda,\nu)\,e^{-\nu/T}\,, \qquad (35)$$
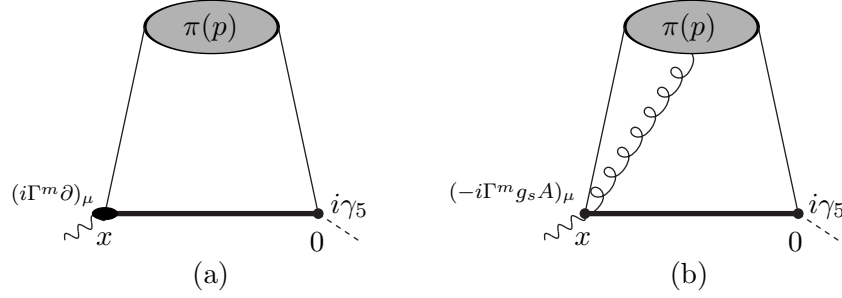
FIG. 2: *The diagrammatic representation of the 2-point vacuum-pion correlation function Eq. (40), where the heavy solid lines represent the heavy quark, and the helical lines a gluon.*

where T is the Borel parameter, and

$$\frac{1}{\pi}\mathrm{Im}F_a^{\mathrm{LO}}(\lambda,\nu) = \frac{f_\pi}{2}\,\Theta(u_0)\left[\frac{2\mu_\pi}{\lambda}\varphi_p(u_0) + \frac{\mu_\pi}{3\lambda}\varphi_\sigma'(u_0) - \frac{4}{\lambda^2}g_2'(u_0)\right], \tag{36}$$

$$\frac{1}{\pi}\mathrm{Im}F_b^{\mathrm{LO}}(\lambda,\nu) = \frac{f_\pi}{2}\,\Theta(u_0)\left[\varphi_\pi(u_0) - \frac{\mu_\pi}{3\lambda}\varphi_\sigma'(u_0) - \frac{4}{\lambda^2}g_1''(u_0) + \frac{4}{\lambda^2}g_2'(u_0)\right]. \tag{37}$$

Here and in the following, $u_0 = 1 - \frac{\nu}{\lambda}$, and the prime denotes derivatives. After integration by parts, we obtain the final sum rules,

$$L_a(v\cdot p) = \frac{f_\pi}{2\,F}\,\mathrm{e}^{2\bar\Lambda/T}\left\{\int_0^\theta \mathrm{d}u\left[\mu_\pi\varphi_p(\bar u) - \frac{\mu_\pi v\cdot p}{3T}\varphi_\sigma(\bar u) + \frac{2}{T}g_2(\bar u)\right]\mathrm{e}^{-2u\,v\cdot p/T}\right.$$
$$\left. - \left[\frac{\mu_\pi}{6}\varphi_\sigma(\bar\theta) - \frac{1}{v\cdot p}g_2(\bar\theta)\right]\mathrm{e}^{-2\theta\,v\cdot p/T}\right\}, \tag{38}$$

$$L_b(v\cdot p) = \frac{f_\pi}{2\,F}\,\mathrm{e}^{2\bar\Lambda/T}\left\{v\cdot p\int_0^\theta \mathrm{d}u\left[\varphi_\pi(\bar u) + \frac{\mu_\pi}{3T}\varphi_\sigma(\bar u) - \frac{4}{T^2}g_1(\bar u) - \frac{2}{v\cdot p\,T}g_2(\bar u)\right]\mathrm{e}^{-2u\,v\cdot p/T}\right.$$
$$\left. + \left[\frac{\mu_\pi}{6}\varphi_\sigma(\bar\theta) - \frac{2}{T}g_1(\bar\theta) + \frac{1}{v\cdot p}\Big(\frac{\mathrm{d}g_1(\bar\theta)}{\mathrm{d}u} - g_2(\bar\theta)\Big)\right]\mathrm{e}^{-2\theta\,v\cdot p/T}\right\}, \tag{39}$$

where $\theta = \mathrm{Min}(1, \frac{\omega_c}{2v\cdot p})$ and $\bar\theta = 1 - \theta$. We see that the LC sum rules become meaningless for processes with a very soft pion which would enhance higher twist contributions.

B. $1/m_Q$ corrections from the effective current

To obtain the $1/m_Q$ corrections from the effective current, we consider the 2-point vacuum-pion correlation function containing a covariant derivative, shown in Fig. 2:

$$E_\mu^m(\lambda,\tilde\omega) = i\int \mathrm{d}^4x\,\mathrm{e}^{ik\cdot x}\,\langle\pi(p)|\,T\big\{\bar u(x)\,(\Gamma^m iD)_\mu\,h_v(x),\,\bar h_v(0)i\gamma_5 d(0)\big\}\,|0\rangle$$
$$= E_a^m(\lambda,\tilde\omega)v_\mu + E_b^m(\lambda,\tilde\omega)\widehat p_\mu, \tag{40}$$

where $m = 1, 3$ is an index introduced for convenience, and

$$(\Gamma^1 iD)_\mu = \gamma_\mu\gamma_\beta iD^\beta, \qquad (\Gamma^3 iD)_\mu = iD_\mu. \tag{41}$$

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The hadronic representation of this 2-point correlation function can be expressed as

$$E_\alpha^{\text{m Hadr}}(\lambda, \tilde{\omega}) = 2F \frac{\mathcal{F}_\alpha^m(\lambda)}{2\bar{\Lambda} - \tilde{\omega}} + resonances,\tag{42}$$

where the $\mathcal{F}_\alpha^m(\lambda)$ have been defined in Eq.(20).

The above correlation function can be calculated directly in HQET. We choose the relevant momentum $\tilde{k}$ to be parallel to v, i.e., $\tilde{k}_\mu = (\tilde{\omega}/2)\, v_\mu$, and work in the fixed-point gauge where $A_\mu(x)$ can be expressed as

$$A_\beta(x) = x^\alpha \int_0^1 \mathrm{d}w\, w\, G_{\alpha\beta}(wx).\tag{43}$$

Eq.(40) then can be given in terms of the above $\mathcal{WF}(2)$s, and the following 3-particle wave functions [8, 16]:

$$\langle \pi(p)|\bar{u}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(wx)d(0)|0\rangle$$
$$= i f_{3\pi}\Big[(p_\alpha p_\mu g_{\beta\nu} - p_\beta p_\mu g_{\alpha\nu}) - (p_\alpha p_\nu g_{\beta\mu} - p_\beta p_\nu g_{\alpha\mu})\Big]\int \mathcal{D}\alpha_i\, \varphi_{3\pi}(\alpha_i)e^{i(\alpha_1 + w\alpha_3)\,px},$$

$$\langle \pi(p)|\bar{u}(x)\gamma_\mu \gamma_5 g_s G_{\alpha\beta}(wx)d(0)|0\rangle$$
$$= f_\pi \Big[p_\beta\Big(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px}\Big) - p_\alpha\Big(g_{\beta\mu} - \frac{x_\beta p_\mu}{px}\Big)\Big]\int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i)e^{i(\alpha_1 + w\alpha_3)\,px}$$
$$+ f_\pi \frac{p_\mu}{px}(p_\alpha x_\beta - p_\beta x_\alpha)\int \mathcal{D}\alpha_i\, \varphi_\parallel(\alpha_i)e^{i(\alpha_1 + w\alpha_3)\,px},$$

$$\langle \pi(p)|\bar{u}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(wx)d(0)|0\rangle$$
$$= i f_\pi \Big[p_\beta\Big(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px}\Big) - p_\alpha\Big(g_{\beta\mu} - \frac{x_\beta p_\mu}{px}\Big)\Big]\int \mathcal{D}\alpha_i\, \tilde{\varphi}_\perp(\alpha_i)e^{i(\alpha_1 + w\alpha_3)\,px}$$
$$+ i f_\pi \frac{p_\mu}{px}(p_\alpha x_\beta - p_\beta x_\alpha)\int \mathcal{D}\alpha_i\, \tilde{\varphi}_\parallel(\alpha_i)e^{i(\alpha_1 + w\alpha_3)\,px},\tag{44}$$

where $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\sigma\tau}G^{\sigma\tau}$, and $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. The wave function $\varphi_{3\pi}(\alpha_i) = \varphi_{3\pi}(\alpha_1, \alpha_2, \alpha_3)$ is twist 3, and $\varphi_\perp$, $\varphi_\parallel$, $\tilde{\varphi}_\perp$ and $\tilde{\varphi}_\parallel$ are twist 4. Using the identities, Eq. (29) and

$$\gamma_\mu \gamma_\alpha \gamma_\beta = \gamma_\mu g_{\alpha\beta} - \gamma_\alpha g_{\mu\beta} + \gamma_\beta g_{\mu\alpha} - i\,\varepsilon_{\mu\alpha\beta\delta}\gamma^\delta \gamma_5,\tag{45}$$

one obtains

$$E_\mu^1(\lambda, \tilde{\omega}) = \frac{f_\pi}{2}\int_0^1 \mathrm{d}u \int_0^\infty \mathrm{d}t\, e^{i\frac{\tilde{\omega}}{2}t}\bigg\{ \overleftarrow{\frac{\mathrm{d}}{\mathrm{d}t}}\Big(\frac{\lambda}{2}\Big[\varphi_\pi(u) + t^2 g_1(u)\Big]\hat{p}_\mu + \mu_\pi \varphi_p(u)v_\mu$$
$$+ it\Big[\frac{\mu_\pi}{12}\lambda\varphi_\sigma(u) - g_2(u)\Big](\hat{p}_\mu - v_\mu)\Big) - i\bar{u}\frac{\lambda}{2}\hat{p}_\mu\Big(\varphi_\pi(u) + t^2 g_1(u)$$
$$+ \mu_\pi \varphi_p(u) + it\Big[\frac{\mu_\pi}{12}\lambda\varphi_\sigma(u) - g_2(u)\Big]\Big) + i\lambda\Big[\frac{\mu_\pi}{4}\varphi_\sigma(u) - it g_1(u)\Big]\hat{p}_\mu$$
$$-2ig_2(u)v_\mu\bigg\}e^{-i\bar{u}\frac{\lambda}{2}t} - \frac{f_\pi}{2}\int \mathcal{D}\alpha_i \int_0^1 \mathrm{d}w\, w \int_0^\infty \mathrm{d}t\, e^{i\frac{\tilde{\omega}}{2}t}\frac{\lambda}{2}t\bigg\{\Big[\lambda\frac{f_{3\pi}}{f_\pi}\varphi_{3\pi}(\alpha_i)$$
$$- \varphi_\parallel(\alpha_i)\Big]\hat{p}_\mu + 2\varphi_\perp(\alpha_i)v_\mu\bigg\}e^{-i(1 - \alpha_1 - w\alpha_3)\frac{\lambda}{2}t},\tag{46}$$

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$$E_\mu^3(\lambda, \tilde{\omega}) = \frac{f_\pi}{2} \int_0^1 du \int_0^\infty dt\, e^{i\frac{\tilde{\omega}}{2}t} \left\{ \frac{\overleftarrow{d}}{dt} \left(\frac{\lambda}{2} \left[\varphi_\pi(u) + t^2 g_1(u) \right] + \mu_\pi \varphi_p(u) \right) v_\mu \right.$$
$$- i\bar{u}\frac{\lambda}{2}\widehat{p}_\mu \left(\frac{\lambda}{2}\left[\varphi_\pi(u) + t^2 g_1(u) \right] + \mu_\pi \varphi_p(u) \right) + \lambda t g_1(u) v_\mu$$
$$\left. + i g_2(u)(\widehat{p}_\mu - v_\mu) \right\} e^{-i\bar{u}\frac{\lambda}{2}t} - \frac{f_\pi}{2} \int \mathcal{D}\alpha_i \int_0^1 dw\, w \int_0^\infty dt\, e^{i\frac{\tilde{\omega}}{2}t}\frac{\lambda}{2}\,t$$
$$\times (v_\mu - \widehat{p}_\mu)\,\varphi_\|(\alpha_i)\, e^{-i(1-\alpha_1-w\alpha_3)\frac{\lambda}{2}t}\,, \tag{47}$$

where terms proportional to $\widehat{p}^2$ are omitted in the chiral limit.

After performing the corresponding Borel transformations, we obtain the sum rules for the effective current corrections:

$$\mathcal{F}_\alpha^m(\lambda) = \frac{e^{2\bar{\Lambda}/T}}{2F}\int_0^{\omega_c} d\nu\, \frac{1}{\pi}\mathrm{Im}E_\alpha^m(\lambda, \nu)\, e^{-\nu/T}\,, \tag{48}$$

where

$$\frac{1}{\pi}\mathrm{Im}E_a^1(\lambda,\nu) = \frac{f_\pi}{2}\Theta(u_0)\Big[\,\bar{u}_0\Big(\mu_\pi\varphi_p(u_0) + \frac{\mu_\pi}{6}\varphi_\sigma'(u_0) - \frac{2}{\lambda}g_2'(u_0)\Big) - \frac{4}{\lambda}g_2(u_0)$$
$$- \frac{2}{\lambda}\int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1} d\alpha_3\, \frac{1}{\alpha_3^2}\, 2\,\varphi_\perp(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\Big]\,, \tag{49}$$

$$\frac{1}{\pi}\mathrm{Im}E_b^1(\lambda,\nu) = \frac{f_\pi}{2}\Theta(u_0)\Big[-\bar{u}_0\Big(\frac{\lambda}{2}\varphi_\pi(u_0) + \mu_\pi\varphi_p(u_0) - \frac{2}{\lambda}g_1''(u_0)\Big) + \frac{\mu_\pi}{3}\varphi_\sigma(u_0)$$
$$+\frac{2}{\lambda}g_2(u_0) - \frac{4}{\lambda}g_1'(u_0) + \frac{2}{\lambda}\int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1} d\alpha_3\, \frac{1}{\alpha_3^2}$$
$$\times \Big(\varphi_\|(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) - \lambda\frac{f_{3\pi}}{f_\pi}\varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\Big)\Big]\,, \tag{50}$$

$$\frac{1}{\pi}\mathrm{Im}E_a^3(\lambda,\nu) = \frac{f_\pi}{2}\Theta(u_0)\Big[\,\bar{u}_0\Big(\frac{\lambda}{2}\varphi_\pi(u_0) + \mu_\pi\varphi_p(u_0) - \frac{2}{\lambda}g_1''(u_0)\Big) - \frac{2}{\lambda}g_2(u_0)$$
$$+ \frac{4}{\lambda}g_1'(u_0) - \frac{2}{\lambda}\int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1} d\alpha_3\, \frac{1}{\alpha_3^2}\, \varphi_\|(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\Big]\,, \tag{51}$$

$$\frac{1}{\pi}\mathrm{Im}E_b^3(\lambda,\nu) = \frac{f_\pi}{2}\Theta(u_0)\Big[-\bar{u}_0\Big(\frac{\lambda}{2}\varphi_\pi(u_0) + \mu_\pi\varphi_p(u_0) - \frac{2}{\lambda}g_1''(u_0)\Big) + \frac{2}{\lambda}g_2(u_0)$$
$$- \frac{4}{\lambda}g_1'(u_0) + \frac{2}{\lambda}\int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1} d\alpha_3\, \frac{1}{\alpha_3^2}\, \varphi_\|(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\Big]\,. \tag{52}$$

Note that one consequence of the equations of motion in Eq. (21), $\mathcal{F}_b^3 = -\mathcal{F}_a^3$, is explicitly satisfied by the sum rules. Combining the above sum rules with Eq. (22) yields,

$$F_6(v \cdot p) = -\frac{f_\pi}{2F} e^{2\bar{\Lambda}/T} v \cdot p \int_0^\theta du \left[\frac{\mu_\pi}{6}\varphi_\sigma(\bar{u}) - \frac{f_{3\pi}}{f_\pi}\int_0^{\bar{u}} d\alpha_1 \int_{\bar{u}-\alpha_1}^{1-\alpha_1} d\alpha_3 \frac{1}{\alpha_3^2} \right.$$
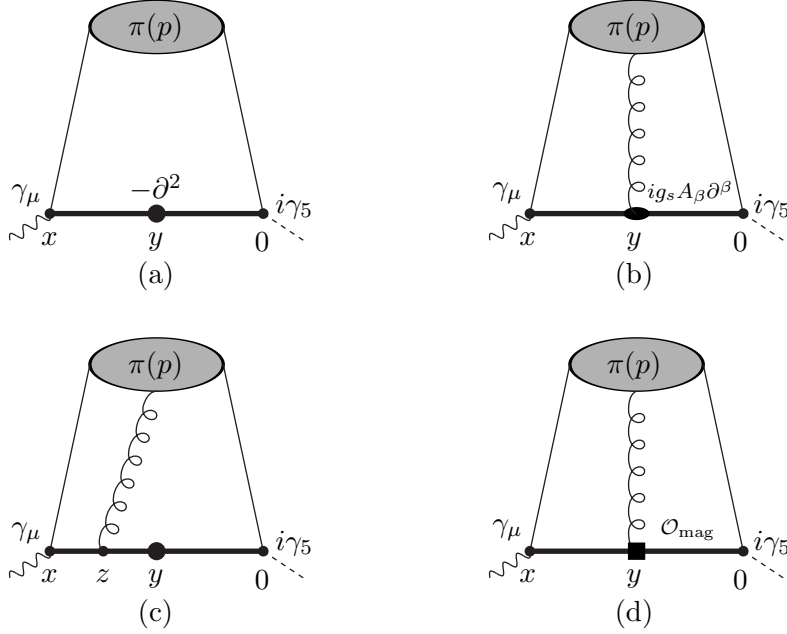$$\left. \times \varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) \right] e^{-2u\,v\cdot p/T}\,. \tag{53}$$

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FIG. 3: *The diagrammatic representation of the leading order vacuum-pion correlation functions with $\mathcal{O}_{\rm kin}$ and $\mathcal{O}_{\rm mag}$ insertions, (see Eq.(54)), where the heavy solid lines represent the heavy quark, and the helical lines a gluon.*

C. $1/m_Q$ corrections from the effective Lagrangian

Finally, we consider the $1/m_Q$ corrections to the correlation function Eq. (23) from the sub-leading operators $O_{\rm kin}$ and $O_{\rm mag}$ in Eq. (4). They are depicted in Fig. 3. One has

$$
\begin{aligned}
F_\mu^{(\frac{1}{m_Q})}(\lambda,\tilde\omega) &= i\int {\rm d}^4 x\, {\rm e}^{ik\cdot x}\, \langle\pi(p)|\, T\Big\{\bar u(x)\gamma_\mu h_v(x),\, i\int {\rm d}^4 y\, \mathcal{L}_1(y)\,, \bar h_v(0)i\gamma_5 d(0)\Big\}\, |0\rangle \\
&= F_a^{(\frac{1}{m_Q})}(\lambda,\tilde\omega)v_\mu + F_b^{(\frac{1}{m_Q})}(\lambda,\tilde\omega)\widehat{p}_\mu\,,
\end{aligned}
\tag{54}
$$

where $\mathcal{L}_1 = \mathcal{O}_{\rm kin} + C_{\rm mag}\mathcal{O}_{\rm mag}$. By denoting $\delta L_\alpha/2m_Q$ as the $1/m_Q$ corrections to L_α, namely

$$
\delta L_\alpha \equiv K_\alpha + C_{\rm mag}\,\mathcal{S}_\alpha\,,
\tag{55}
$$

the hadronic representation of this correlator can be written as [17]

$$
F_\alpha^{(\frac{1}{m_Q})\,{\rm Hadr}}(\lambda,\tilde\omega) = \frac{2F\,\delta F\,L_\alpha(\lambda)}{2\bar\Lambda - \tilde\omega} + \frac{2F\,\delta L_\alpha(\lambda)}{2\bar\Lambda - \tilde\omega} - \frac{4F\,\delta\bar\Lambda\,L_\alpha(\lambda)}{(2\bar\Lambda - \tilde\omega)^2} + resonances\,,
\tag{56}
$$

where $\delta F/2m_Q$ and $\delta\bar\Lambda/2m_Q$ are the $1/m_Q$ corrections for F and $\bar\Lambda$ [18, 19], respectively.

We now calculate the sub-leading correlation function Eq. (54) in HQET, starting from the relation

$$
\partial^\beta A_\beta \cong \mathcal{WF}(4){\rm s} + \mathcal{O}(\alpha_s)\,\mathcal{WF}(2){\rm s}\,,
\tag{57}
$$

where the symbol $\cong$ indicates that this equation holds at the level of matrix elements in terms of LC wave functions. This relation can be found by considering Eq.(43) and the

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equation of motion for the gluon fields, $\partial^\alpha G^a_{\alpha\beta} = -f^{abc} A^{b\alpha} G^c_{\alpha\beta} + g_s \bar{h}_v v_\beta T^a h_v$. One can drop all the terms containing $\partial^\beta A_\beta$ or $A^\beta A_\beta$, because contributions from wave functions with more than 3 particles and from order α_s contributions are physically quite small [8]. Therefore, we have

$$
F_\mu^{(\frac{1}{m_Q})}(\lambda, \tilde\omega) = i \int \mathrm{d}^4 x \, \mathrm{e}^{ik\cdot x} \, i \int \mathrm{d}^4 y \, \langle \pi(p)| \, T\Big\{ \bar{u}(x)\gamma_\mu h_v(x), \, \bar{h}_v(y)\Big[-\partial^2 + 2\, i\, g_s A_\beta \partial^\beta
$$
$$
+ C_{\mathrm{mag}} \frac{g_s}{2} \sigma_{\alpha\beta} G^{\alpha\beta} \Big] h_v(y), \, \bar{h}_v(0) i\gamma_5 d(0) \Big\} |0\rangle \,. \quad (58)
$$

Here the higher-order terms in the heavy quark propagator may be included, as displayed in Fig. 3 (c). However, direct evaluation shows their contribution to be zero. Using the identities Eqs. (29) and (45) together with

$$
\gamma_\mu \sigma_{\alpha\beta} = i\,(g_{\mu\alpha}\gamma_\beta - g_{\mu\beta}\gamma_\alpha) + \varepsilon_{\mu\alpha\beta\delta}\gamma^\delta\gamma_5 \,, \quad (59)
$$

$$
\gamma_\mu \sigma_{\alpha\beta}\gamma_\nu = i\,(g_{\mu\alpha}g_{\beta\nu} - g_{\mu\beta}g_{\alpha\nu}) + (\sigma_{\beta\nu}g_{\mu\alpha} - \sigma_{\alpha\nu}g_{\mu\beta}) - \varepsilon_{\mu\alpha\beta\nu}\gamma_5 + i\,\varepsilon_{\mu\alpha\beta\delta}g^{\delta\rho}\sigma_{\rho\nu}\gamma_5 \,, \quad (60)
$$

and after some algebraic manipulations, we obtain

$$
F_\mu^{(\frac{1}{m_Q})}(\lambda, \tilde\omega) = \frac{f_\pi}{2} \int_0^1 \mathrm{d}u \int_0^\infty \mathrm{d}t\, \mathrm{e}^{i\frac{\tilde\omega}{2}t} \Bigg\{ \frac{\overleftarrow{\mathrm{d}^2}}{\mathrm{d}t^2} \Big[\frac{\lambda}{2}\Big[t\varphi_\pi(u) + t^3 g_1(u) \Big]\hat{p}_\mu + \mu_\pi t\varphi_p(u)v_\mu
$$
$$
+ it^2\Big[\frac{\mu_\pi}{12}\lambda\varphi_\sigma(u) - g_2(u) \Big](\hat{p}_\mu - v_\mu) \Big] + \frac{\overleftarrow{\mathrm{d}}}{\mathrm{d}t}\Big[2it\Big[\frac{\mu_\pi}{12}\lambda\varphi_\sigma(u) - g_2(u) \Big](\hat{p}_\mu - v_\mu) \Big]
$$
$$
+ 2\lambda t^2 g_1(u)\hat{p}_\mu - i\bar{u}\lambda\Big(\frac{\lambda}{2}\Big[t\varphi_\pi(u) + t^3 g_1(u) \Big]\hat{p}_\mu + \mu_\pi t\varphi_p(u)v_\mu
$$
$$
+ it^2\Big[\frac{\mu_\pi}{12}\lambda\varphi_\sigma(u) - g_2(u) \Big](\hat{p}_\mu - v_\mu) \Big) \Big] + \Big(\lambda t g_1(u) + 2i g_2(u)
$$
$$
- i\bar{u}\lambda\Big[\lambda t^2 g_1(u) - it g_2(u) \Big]\Big)\hat{p}_\mu \Bigg\}\, \mathrm{e}^{-i\bar{u}\frac{\lambda}{2}t} - f_\pi\int \mathcal{D}\alpha_i \int_0^1 \mathrm{d}s\, s \int_0^1 \mathrm{d}w\, w \int_0^\infty \mathrm{d}t\, \mathrm{e}^{i\frac{\tilde\omega}{2}t}
$$
$$
\times \frac{\lambda}{2}t\,\hat{p}_\mu \Big\{ 2\Big[\varphi_\parallel(\alpha_i) + \varphi_\perp(\alpha_i) \Big] - i(1 - \alpha_1 - sw\alpha_3)\frac{\lambda}{2}t\,\varphi_\parallel(\alpha_i) \Big\}\, \mathrm{e}^{-i(1-\alpha_1 - sw\alpha_3)\frac{\lambda}{2}t}
$$
$$
+ C_{\mathrm{mag}}\frac{f_\pi}{2}\int \mathcal{D}\alpha_i \int_0^1 \mathrm{d}w \int_o^\infty \mathrm{d}t\, \mathrm{e}^{i\frac{\tilde\omega}{2}t}\frac{\lambda}{2}t\Big\{ -\Big[2\tilde\varphi_\perp(\alpha_i) - \tilde\varphi_\parallel(\alpha_i) + 2\varphi_\perp(\alpha_i) \Big]\hat{p}_\mu
$$
$$
+ 2\varphi_\perp(\alpha_i)\, v_\mu \Big\}\, \mathrm{e}^{-i(1-\alpha_1 - w\alpha_3)\frac{\lambda}{2}t} \,. \quad (61)
$$

Having all necessary results at hand, we obtain the sum rules resulting from order $1/m_Q$ power corrections to the effective Lagrangian:

$$
\delta L_\alpha(\lambda) = \frac{1}{2F}\int_0^{\omega_c} \mathrm{d}\nu\, \frac{1}{\pi}\,\mathrm{Im}\, F_\alpha^{(\frac{1}{m_Q})}(\lambda, \nu)\, \mathrm{e}^{(2\bar\Lambda - \nu)/T} + \Big[2\,\delta\bar\Lambda/T - \delta F \Big] L_\alpha(\lambda) \,, \quad (62)
$$

where the sub-leading HQET spectral density functions are given by

$$
\frac{1}{\pi}\mathrm{Im} F_a^{(\frac{1}{m_Q})}(\lambda, \nu) = \frac{f_\pi}{2}\Theta(u_0)\Big\{ \bar{u}_0^2\Big[\mu_\pi\varphi_p'(u_0) + \frac{\mu_\pi}{6}\varphi_\sigma''(u_0) - \frac{2}{\lambda}g_2''(u_0) \Big] - 2\bar{u}_0\Big[\mu_\pi\varphi_p(u_0)
$$
$$
+ \frac{\mu_\pi}{6}\varphi_\sigma'(u_0) - \frac{2}{\lambda}g_2'(u_0) \Big] \Big\} \,, \quad (63)
$$

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$$\frac{1}{\pi}\mathrm{Im}F_b^{(\frac{1}{m_Q})}(\lambda,\nu) = \frac{f_\pi}{2}\Theta(u_0)\Bigg\{ \bar{u}_0^2\Big[\frac{\lambda}{2}\varphi_\pi'(u_0) - \frac{\mu_\pi}{6}\varphi_\sigma''(u_0) + \frac{2}{\lambda}g_2''(u_0) - \frac{2}{\lambda}g_1'''(u_0)\Big]$$

$$- 2\bar{u}_0\Big[\frac{\lambda}{2}\varphi_\pi(u_0) - \frac{\mu_\pi}{6}\varphi_\sigma'(u_0) + \frac{2}{\lambda}g_2'(u_0) - \frac{6}{\lambda}g_1''(u_0)\Big] + \frac{8}{\lambda}g_2(u_0) - \frac{12}{\lambda}g_1'(u_0)$$

$$+ \frac{8}{\lambda}\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\mathrm{d}\alpha_3\frac{1}{\alpha_3^2}\Big[\Big(\frac{\alpha_3}{u_0-\alpha_1}-1\Big)\varphi_\|(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)$$

$$- \ln\Big(\frac{\alpha_3}{u_0-\alpha_1}\Big)\big[\varphi_\| + \varphi_\perp\big](\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\Big]\Bigg\}. \tag{64}$$

It is interesting to note that the operator $\mathcal{O}_{mag}$ in fact does not contribute to the HQET spectral density functions even at order $1/m_Q$.

IV. NUMERICAL ANALYSIS

Let us now analyze the sum rules numerically. We use the pion wave functions collected and carefully discussed in Ref. [8, 16]. They read,

$$\varphi_\pi(u,\mu) = 6u\bar{u}\Big\{1 + a_2(\mu)\frac{3}{2}[5(u-\bar{u})^2-1] + a_4(\mu)\frac{15}{8}[21(u-\bar{u})^4 - 14(u-\bar{u})^2 + 1]\Big\},$$

$$\varphi_p(u,\mu) = 1 + \tilde{B}_2(\mu)\frac{1}{2}[3(u-\bar{u})^2-1] + \tilde{B}_4(\mu)\frac{1}{8}[35(u-\bar{u})^4 - 30(u-\bar{u})^2 + 3],$$

$$\varphi_\sigma(u,\mu) = 6u\bar{u}\Big\{1 + \tilde{C}_2(\mu)\frac{3}{2}[5(u-\bar{u})^2-1] + \tilde{C}_4(\mu)\frac{15}{8}[21(u-\bar{u})^4 - 14(u-\bar{u})^2 + 1]\Big\},$$

$$g_1(u,\mu) = \frac{5}{2}\delta^2(\mu)\bar{u}^2u^2 + \frac{1}{2}\varepsilon(\mu)\delta^2(\mu)[\bar{u}u(2+13\bar{u}u) + 10u^3\ln u(2-3u+\frac{6}{5}u^2)$$

$$+ 10\bar{u}^3\ln\bar{u}(2-3\bar{u}+\frac{6}{5}\bar{u}^2)],$$

$$g_2(u,\mu) = \frac{10}{3}\delta^2(\mu)\bar{u}u(u-\bar{u}),$$

$$\varphi_{3\pi}(\alpha_i,\mu) = 360\alpha_1\alpha_2\alpha_3^2\Big[1 + \omega_{1,0}(\mu)\frac{1}{2}(7\alpha_3-3) + \omega_{2,0}(\mu)(2-4\alpha_1\alpha_2-8\alpha_3+8\alpha_3^2)$$

$$+ \omega_{1,1}(\mu)(3\alpha_1\alpha_2 - 2\alpha_3 + 3\alpha_3^2)\Big],$$

$$\varphi_\perp(\alpha_i,\mu) = 30\delta^2(\mu)(\alpha_1-\alpha_2)\alpha_3^2[\frac{1}{3} + 2\varepsilon(\mu)(1-2\alpha_3)],$$

$$\varphi_\|(\alpha_i,\mu) = 120\delta^2(\mu)\varepsilon(\mu)(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3,$$

$$\tilde{\varphi}_\perp(\alpha_i,\mu) = 30\delta^2(\mu)\alpha_3^2(1-\alpha_3)[\frac{1}{3} + 2\varepsilon(\mu)(1-2\alpha_3)],$$

$$\tilde{\varphi}_\|(\alpha_i,\mu) = -120\delta^2\alpha_1\alpha_2\alpha_3[\frac{1}{3} + \varepsilon(\mu)(1-3\alpha_3)], \tag{65}$$

where μ is the renormalization scale. The scale dependence of these wave functions is given by perturbative QCD.

The light-cone sum rules obtained in HQET, and the pion wave functions in Eq. (65) depend on the subtraction point μ. However, in the lowest order of α_s, the value of the scale μ is ambiguous. Within HQET, a reasonable choice is $\mu \simeq 2\bar{\Lambda}$. Thus we set the

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renormalization point at $\mu_0 = 1$ GeV. In this case, $\mu_\pi(\mu_0) = 1.65$ GeV, and $r(\mu_0) = 1.08$. The coefficients appearing in the above equation are then taken as [8]

$$a_2(\mu_0) = 0.44\,, \quad a_4(\mu_0) = 0.25\,, \quad \tilde{B}_2(\mu_0) = 0.48\,, \quad \tilde{B}_4(\mu_0) = 1.15\,,$$
$$\tilde{C}_2(\mu_0) = 0.10\,, \quad \tilde{C}_4(\mu_0) = 0.067\,, \quad \delta^2(\mu_0) = 0.2\,\text{GeV}^2\,, \quad \varepsilon(\mu_0) = 0.5\,,$$
$$f_{3\pi}(\mu_0) = 0.0035\,\text{GeV}^2\,, \quad \omega_{1,0}(\mu_0) = -2.88\,, \quad \omega_{2,0}(\mu_0) = 10.5\,, \quad \omega_{1,1}(\mu_0) = 0\,. \tag{66}$$

The pion decay constant $f_\pi = 0.132$ GeV is taken from experiment. The parameters for heavy mesons are obtained from HQET sum rules [13, 18, 19]: $\bar{\Lambda} = 0.57$ GeV, $F(1\text{GeV}) = 0.46$ GeV$^{3/2}$, $\delta\bar{\Lambda} = -0.35$ GeV^2 and $\delta F = -1.92$ GeV.

A. Universal functions

In order to make the sum rules meaningful, we must first check the existence of sum rule windows, which are roughly given by $\Lambda_{\text{QCD}} < T < 2\bar{\Lambda}$. The lower and upper limit will be obtained by the physical requirement that the Borel parameter T must be large enough to ensure that the higher-twist wave function contributions are suppressed, and at the same time small enough in order to make the resonance contribution not too large. To be specific, for the typical threshold values $\omega_c \simeq 2$ GeV, we find that, setting the Borel parameter to 0.60 GeV $\leq T \leq 1.00$ GeV for the sum rules L_a and L_b, the twist-4 wave functions give contributions less than 17% and 3%, while the resonance contributions are lower than 27% and 6%, respectively. The two sum rules are quite stable in this region, as shown in Fig. 4(a) and (b) where $v \cdot p$ is fixed at 2.0 GeV. Nevertheless, our numerical calculations show that the stability of the sum rules is not very sensitive to a change of ω_c. In order to be consistent with HQET sum rule calculations, we adopt the same stability region of ω_c as that from the $\bar{\Lambda}$ sum rule [13, 18], i.e., $\omega_c = 2.0 \pm 0.3$ GeV. Fig. 5(a) and (b) present the final results for $L_a(v \cdot p)$ and $L_b(v \cdot p)$, respectively, where the central value of the Borel parameter $T = 0.80$ GeV is used. Note that the LC-HQET sum rules are meaningless in the soft pion region as mentioned above. To stay away from the soft pion region, we take $v \cdot p > 1.2$ GeV.

For the convenience of further applications, we parametrize the results by the following formulae in the region 1.2 GeV $< v \cdot p < 2.64$ GeV,

$$L_a^{\text{Fit}}(v \cdot p) = \frac{1}{a_0 + a_1\,v \cdot p + a_2\,(v \cdot p)^2}\,, \tag{67}$$

$$L_b^{\text{Fit}}(v \cdot p) = b_0 + b_1\,v \cdot p + \frac{b_2}{v \cdot p}\,. \tag{68}$$

Best fit values in which the maximal errors are less than 2% yields the following values of the parameters:

ω_c [GeV]	a_0 [GeV$^{-1/2}$]	a_1 [GeV$^{-3/2}$]	a_2 [GeV$^{-5/2}$]	b_0 [GeV$^{1/2}$]	b_1 [GeV$^{-1/2}$]	b_2 [GeV$^{3/2}$]
2.3	-0.78	4.00	0.891	0.403	-0.0221	-0.0385
2.0	-3.03	6.71	0.286	0.409	-0.0263	-0.0448
1.7	-6.25	10.9	-0.743	0.320	-0.0049	0.0343

Next we consider the $1/m_Q$ corrections from the effective current. In the relevant universal function F_6, only two twist-3 wave functions appear. Thus, in order to determine the
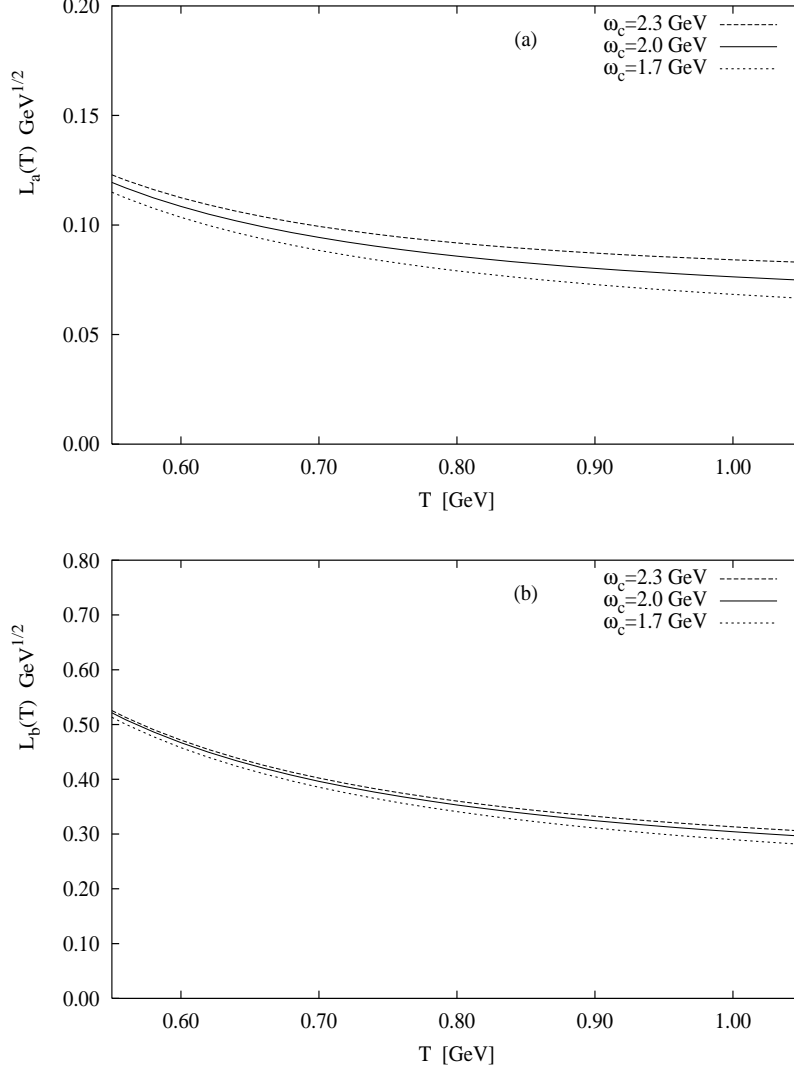
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FIG. 4: *Variation of the sum rule predictions on $L_{a,b}$ with the Borel parameter T at various values of the continuum threshold parameter ω_c.*

sum rule window, especially its lower limit, one may consult the sum rules for $\mathcal{F}^1_{a,b}$ that will contribute to the form factors directly while the renormalization-group effects are not considered. For threshold values of $\omega_c \simeq 1.8$ GeV, requiring that both the twist-4 wave functions and the resonance contributions do not exceed 40%, the sum rule window can be determined to be 0.50 GeV $\leq T \leq 0.70$ GeV, in which we find that the resonance contributions for F_6 are less than 8%. The numerical results for the universal function F_6 as a function of the Borel parameter T are shown in Fig. 6, where $v \cdot p$ is fixed to be 2.0 GeV. Fig. 7 presents the results for $F_6(v \cdot p)$, where the central value $T = 0.60$ GeV is taken for the Borel parameter.

The results can be parametrized by the formula below valid in the region 1.2 GeV $< v \cdot p < 2.64$ GeV, with maximal errors less than 3%,

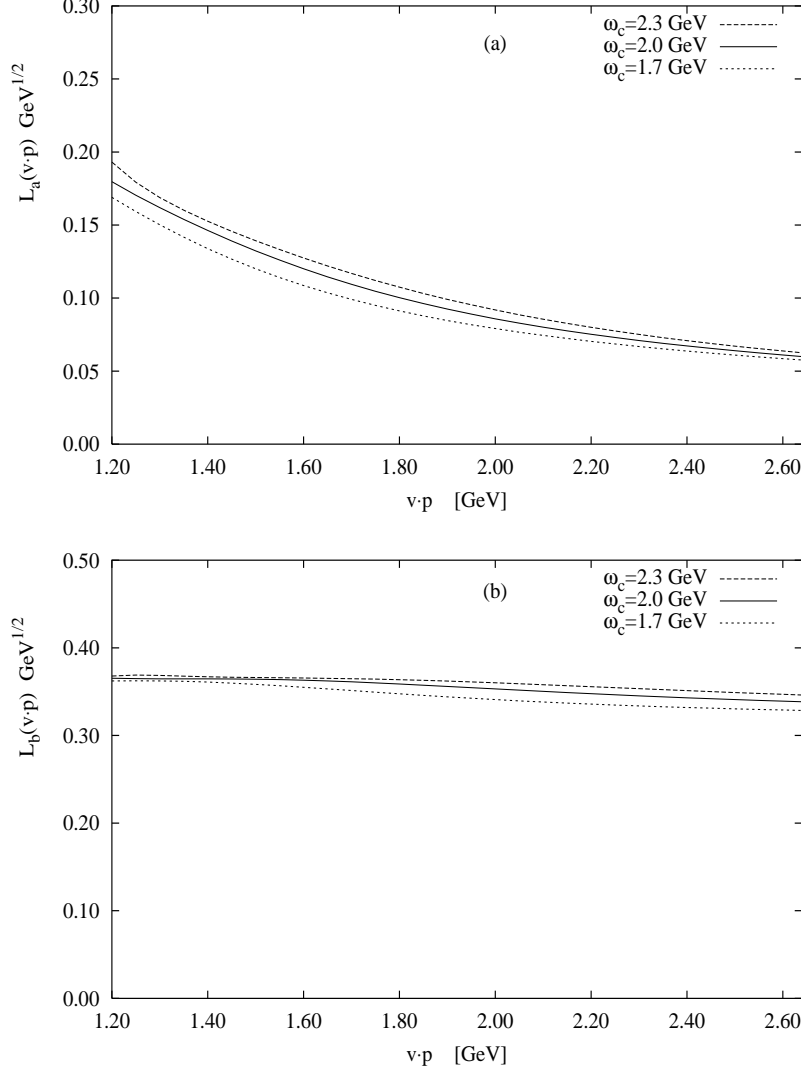$$F_6^{\text{Fit}}(v \cdot p) \; = \; \frac{1}{c_0 + c_1 \, v \cdot p + c_2 \, (v \cdot p)^2} \, . \tag{69}$$

FIG. 5: *The universal functions $L_{a,b}(v \cdot p)$ obtained from LC-HQET sum rules at various values of the continuum threshold parameter ω_c.*

The parameters c_i are given by

ω_c [GeV]	c_0 [GeV$^{-3/2}$]	c_1 [GeV$^{-5/2}$]	c_2 [GeV$^{-7/2}$]
2.1	-112	30.5	-4.93
1.8	-86.1	1.54	2.22
1.5	-57.6	-34.0	11.7

We next analyze the sum rules resulting from the $1/m_Q$ insertions due to the power corrections to the effective Lagrangian. We find that, for the sum rule for δL_a, the twist-4 wave function contributions are very small everywhere in the physically appropiate region of T, but the resonance contributions grow rapidly as T becomes large. Requiring the latter contributions to be less than 40% yields $T \leq 0.55$ GeV. Consequently, we set the range of the Borel parameter at 0.35 GeV $\leq T \leq 0.55$ GeV. The results of the sum rule for δL_b are very similar to the leading order sum rules. In the sum rule window 0.60 GeV $\leq T \leq 1.00$
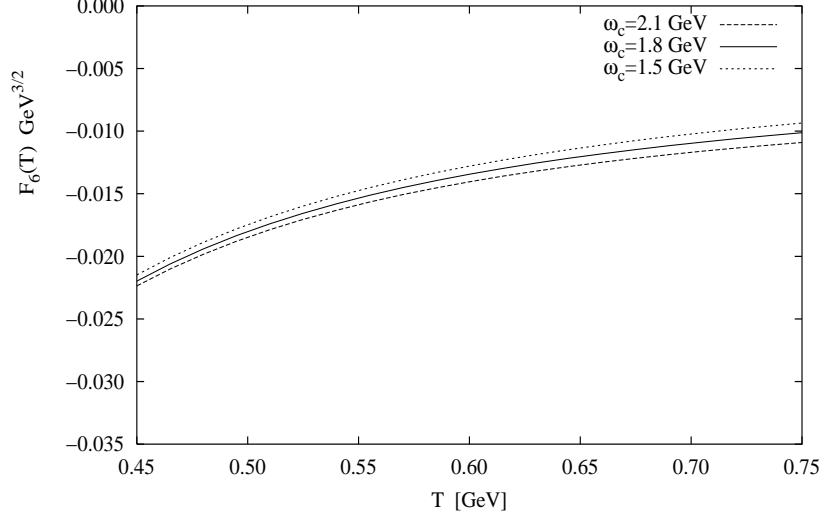
FIG. 6: *Dependence of the sum rule predictions of F_6 on the Borel parameter T at various values of the continuum threshold parameter ω_c.*
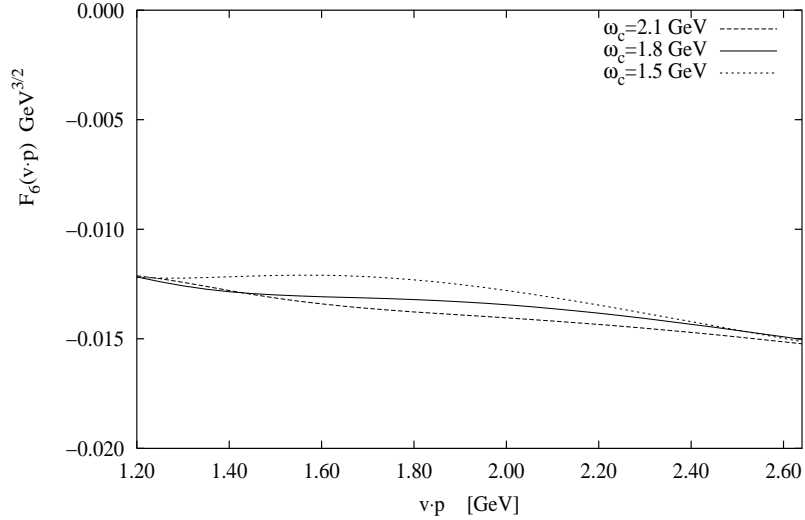

FIG. 7: *The sub-leading universal function $F_6(v \cdot p)$ obtained from the LC-HQET sum rules at various values of the continuum threshold parameter ω_c.*

GeV and for threshold values $\omega_c \simeq 1.8$ GeV, the twist-4 wave function contributions are less than 11% and the resonance contribution does not exceed 18%. The stability of the sum rules for δL_α ($\alpha = a$, b) with regard to variations of the Borel parameter T are shown in Fig. 8(a) and (b), where $v \cdot p$ is fixed at 2.0 GeV. Fig. 9(a) and (b) present our final results for $\delta L_a(v \cdot p)$ and $\delta L_b(v \cdot p)$, respectively, where the corresponding central value of the Borel parameter $T = 0.45$ and 0.80 GeV are used. It should be mentioned that, in these two sub-leading universal functions, the sub-leading HQET spectral density functions give small and negative contributions to the form factors, while the corrections coming from the δF and $\delta \bar{\Lambda}$ contributions are large, and positive when the sum is taken (for δL_a, the proporation is about $-1 : 2$, and δL_b about $-1 : 5$).

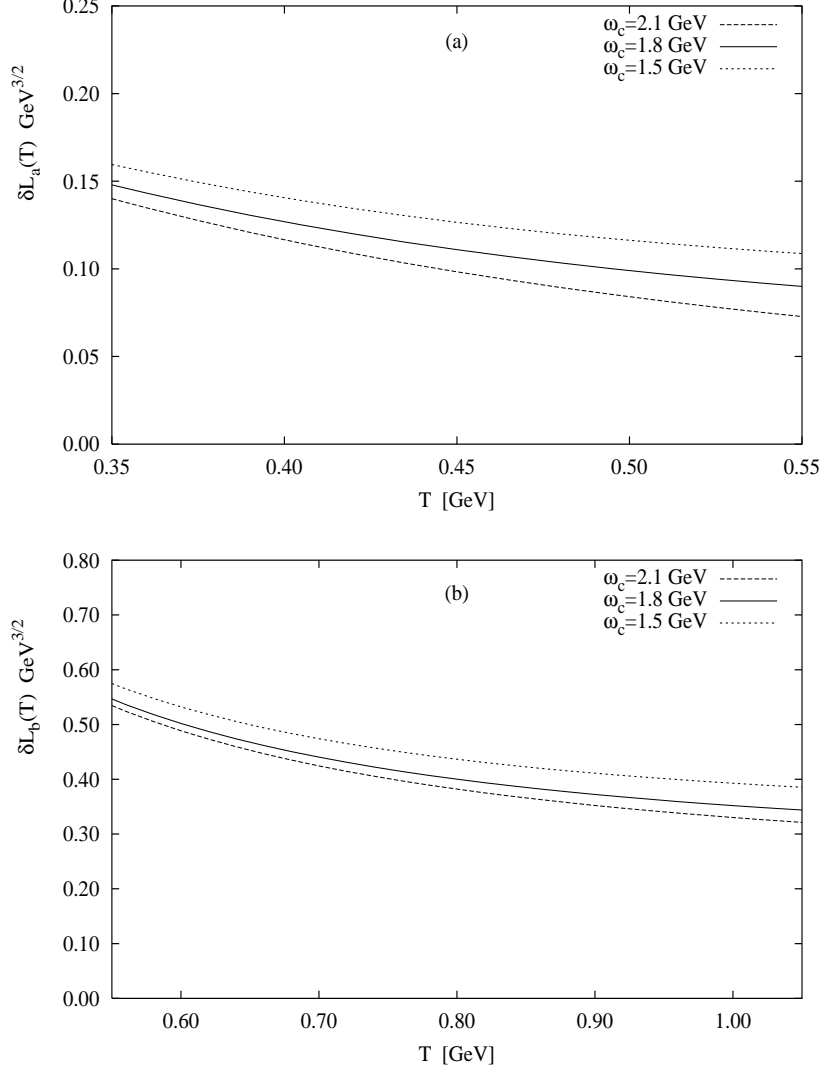We have parametrized these results by the following formulae in the region 1.2 GeV <

FIG. 8: *Dependence of the sum rule predictions $\delta L_{a,b}$ on the Borel parameter T at various values of the continuum threshold parameter ω_c.*

$v \cdot p < 2.64$ GeV,

$$\delta L_a^{\text{Fit}}(v \cdot p) \;=\; \frac{1}{\tilde{a}_0 + \tilde{a}_1\, v \cdot p + \tilde{a}_2\, (v \cdot p)^2}\,, \tag{70}$$

$$\delta L_b^{\text{Fit}}(v \cdot p) \;=\; \tilde{b}_0 + \tilde{b}_1\, v \cdot p + \frac{\tilde{b}_2}{v \cdot p}\,. \tag{71}$$

The best fitting parameters are

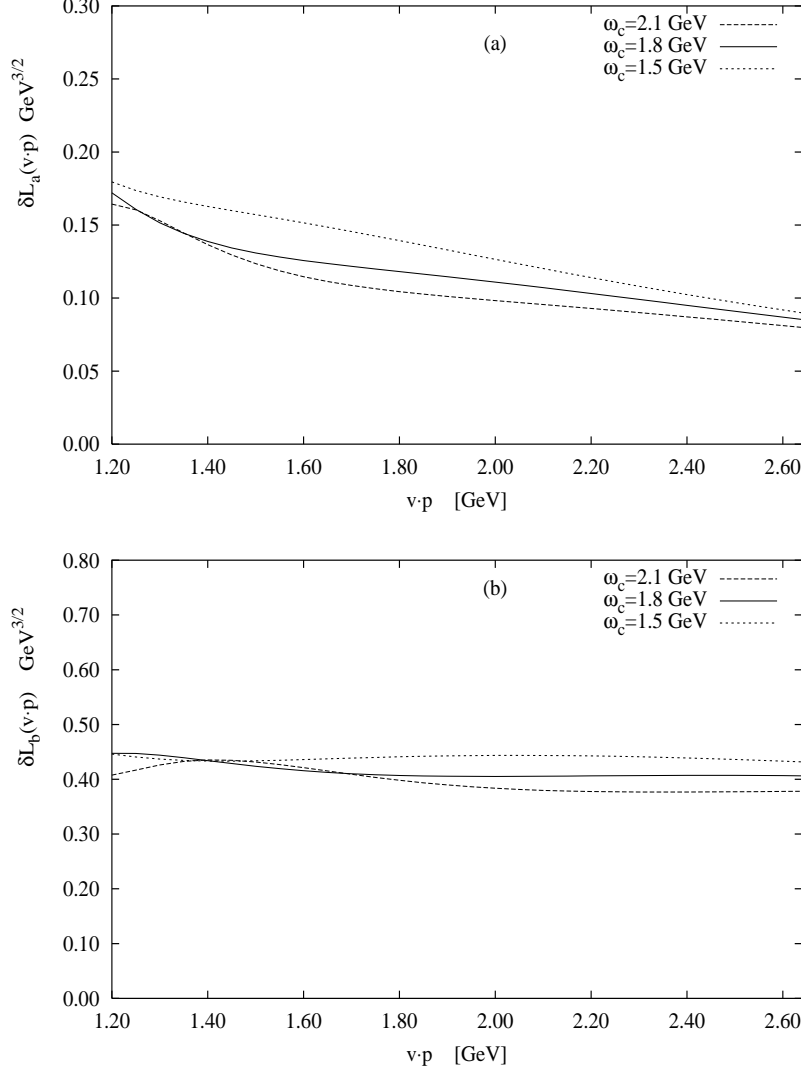| ω_c [GeV]	$\tilde{a}_0$ [GeV$^{-3/2}$]	$\tilde{a}_1$ [GeV$^{-5/2}$]	$\tilde{a}_2$ [GeV$^{-7/2}$]	$\tilde{b}_0$ [GeV$^{3/2}$]	$\tilde{b}_1$ [GeV$^{1/2}$]	$\tilde{b}_2$ [GeV$^{5/2}$]
2.1	-4.56	10.6	-1.59	0.319	0.0059	0.113
1.8	1.62	3.57	0.088	0.129	0.0623	3.00
1.5	6.05	-2.16	1.55	0.539	-0.0294	-0.077

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FIG. 9: *The sub-leading universal functions $\delta L_{a,b}(v \cdot p)$ obtained from LC-HQET sum rules at various values of the continuum threshold parameter ω_c.*

where the maximal fitting errors are less than 5%.

Finally we remark on the errors in the sum rules. The uncertainties of the continuum threshold parameter ω_c induce errors (from both the LC-HQET sum rules themselves and the sum rules of $\bar{\Lambda}$ and F [13]) less than $\pm 10\%$ in the final sum rule results. However, one should keep in mind that the sum rule method typically has a $\pm (10 \sim 30)\%$ uncertainty resulting from the duality assumption, uncertainties in the wave functions and other input parameters.

B. Form factors to order $1/m_Q$

Having obtained the leading and the relevant sub-leading universal functions, we can construct the form factors $f_{1,2}(v \cdot p)$ from Eq. (18) for the $B \to \pi$ transition to order $1/m_b$. The results are shown in Fig. 10 (a) and (b), where we take $m_b = 4.7$ GeV. For comparison,

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the leading order results corresponding to $\omega_c = 2.0$ GeV are also shown in this figure. Here the sub-leading universal functions roughly give a $11 \sim 23\%$ and $5 \sim 12\%$ enhancement for the form factors $f_1^{B \to \pi}(v \cdot p)$ and $f_2^{B \to \pi}(v \cdot p)$, respectively.
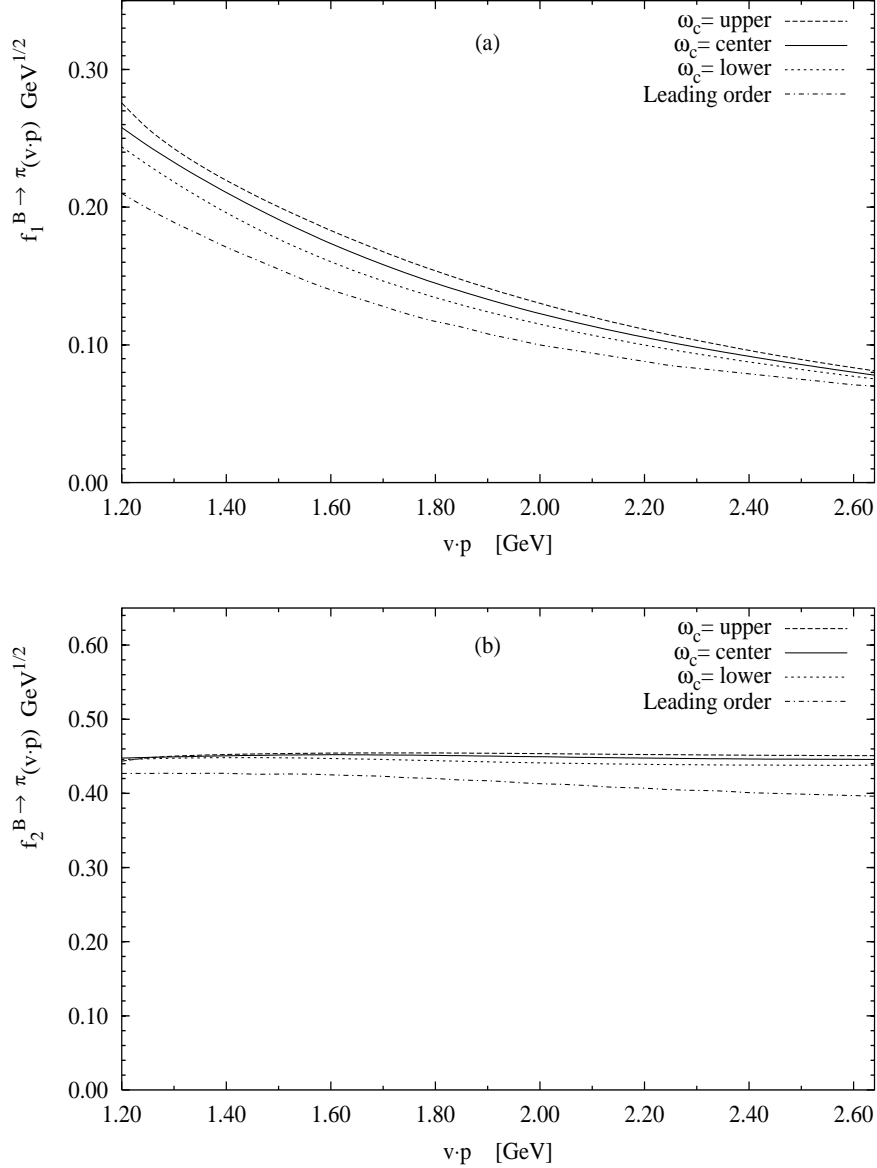
FIG. 10: *The form factors* $f_{1,2}^{B \to \pi}(v \cdot p)$ *obtained from the LC-HQET sum rules at various values of the continuum threshold parameter* ω_c.

V. COMPARISON WITH OTHER MODEL CALCULATIONS, DISCUSSION AND SUMMARY

It is interesting to compare our results with those of other approaches. First let us look at the soft pion limit. The heavy meson chiral perturbation theory (HMχPT) [20] (see also [21]) describes $f_1(v \cdot p)$ and $f_2(v \cdot p)$ in a single pole form in the soft pion region. Moreover, the

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$1/m_Q$ corrections do not change this behaviours [6, 22]. In the case of $B \to \pi$ transitions, they can be expressed by

$$f_{1\,\text{HM}\chi\text{PT}}^{B\to\pi}(v \cdot p) = \frac{C_1 F}{2f_\pi}\left(1 + \frac{\delta F}{2m_b}\right)\left[1 - \frac{g\,v \cdot p}{v \cdot p + \Delta_B}\right],\tag{72}$$

$$f_{2\,\text{HM}\chi\text{PT}}^{B\to\pi}(v \cdot p) = \frac{C_1 F}{2f_\pi}\left(1 + \frac{\delta F}{2m_b}\right)\frac{g\,v \cdot p}{v \cdot p + \Delta_B},\tag{73}$$

where $\Delta_B \simeq m_{B^*} - m_B = 0.045$ GeV, $g \sim 0.3$ is the coupling of the pion to the heavy meson. It is easy to see that our result for $f_1^{B\to\pi}(v \cdot p)$ does not match to the HMχPT result. Concerning $f_2^{B\to\pi}(v \cdot p)$ our sum rule result Eq. (39) can be seen to reasonably well match on to the soft pion result considering the uncertainties in the values of F and g. By taking $g = 0.28$, one can find that the extrapolation of HMχP to large $v \cdot p$ matches quite well with the sum rule calculations at intermediate pion energies as shown in Fig 11 (a). For practical purposes, we use the following Gaussian-type function to make a smooth connection between the sum rule result of $f_1^{B\to\pi}(v \cdot p)$ and that of HMχPT,

$$f_{1\,\text{G}}^{B\to\pi}(v \cdot p) = \frac{C_1 F}{2f_\pi}\left(1 + \frac{\delta F}{2m_b}\right)\left[g_0 + g_1 \mathrm{e}^{-g_2(v\cdot p - \delta_B)^2}\right],\tag{74}$$

$$(\text{for } 0.25 \text{ GeV} < v \cdot p < 1.2 \text{ GeV})$$

where $g_0 = 0.125$, $g_1 = 0.751$, $g_2 = 3.0$ GeV^{-2} and $\delta_B = 0.23$ GeV were obtained by matching both sides. This is plotted in Fig. 11 (b). It should be mentioned that the matching results obtained here correspond to the case of central values of ω_c. For further support of our matching procedure we have included in the lattice NRQCD results from the JLQCD collaboration [23]. They can be seen to be well consistent with our matching result.

The large pion energy limit is another interesting limit to consider because the large energy effective theory (LEET) [24, 25] provides some model-independent information on this kinematical region. In the $B \to \pi$ semileptonic decay, the light non-spectator quark gets a large amount of energy from the decay of the bottom quark. The light spectator quark system interacts with the energetic quark mainly at the energy scale Λ_{QCD} which is basically fixed by the size of hadrons. New symmetries appear in the limit of $v \cdot p / \Lambda_{\text{QCD}} \to \infty$. They are subject to corrections by hard gluon exchange [26]. In the leading order of the heavy quark expansion our results are

$$L_a(\infty) = 0\,, \quad L_b(\infty) = \frac{f_\pi}{F}T\phi_\pi(1)\,I_0\,,\tag{75}$$

where

$$I_i = \frac{1}{4}e^{2\bar{\Lambda}/T}\int_0^{\omega_c/T}\mathrm{d}x\,x^i e^{-x}\,,\tag{76}$$

This agrees with LEET. For the $1/m_Q$ corrections, our results are

$$F_6(\infty) = -\frac{f_\pi}{F}\frac{\mu_\pi}{6}\phi_\sigma(1)\,I_0\,,$$

$$\delta L_a(\infty) = 0\,,\tag{77}$$

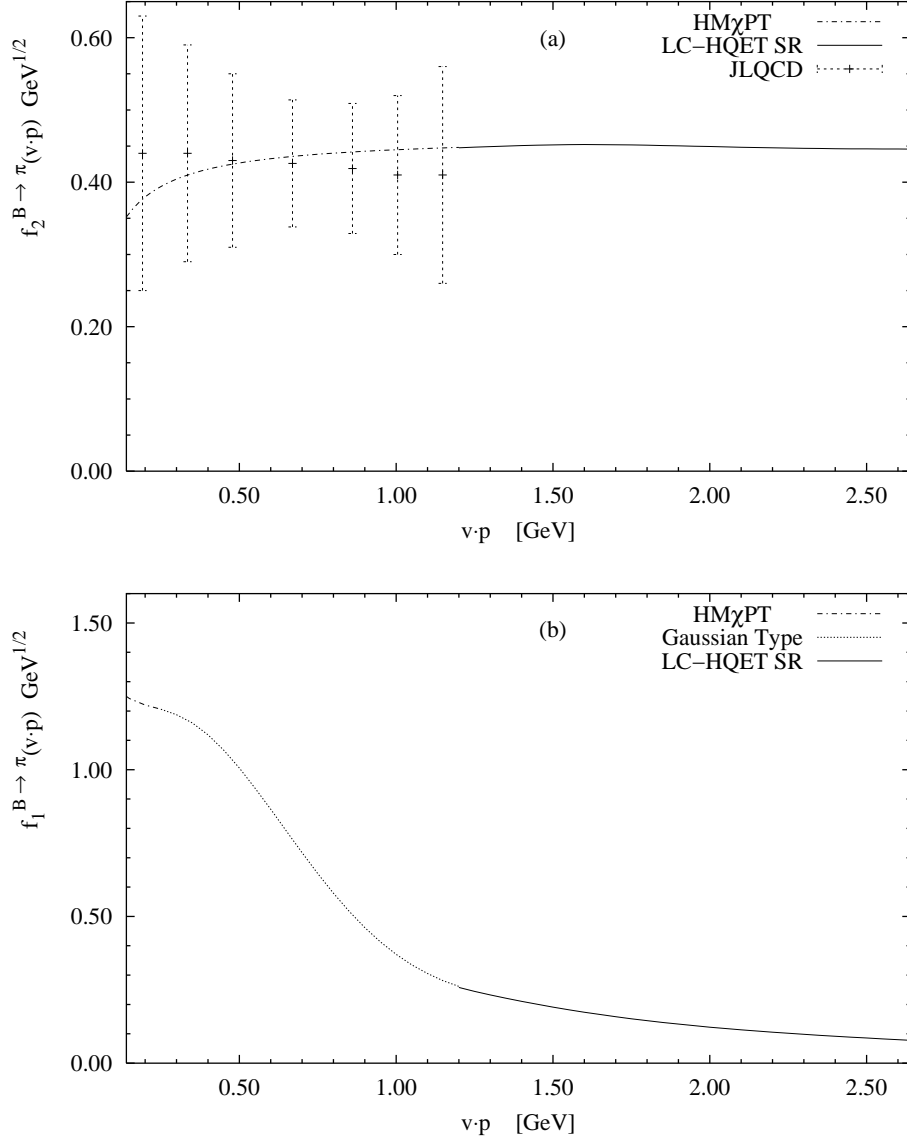$$\delta L_b(\infty) = \frac{f_\pi}{F}T\left[(2\delta\bar{\Lambda}/T - \delta F)\,I_0 - T\,I_1)\right]\phi_\pi(1)\,.$$

FIG. 11: The form factors $f_{1,2}^{B \to \pi}(v \cdot p)$ as obtained from LC-HQET sum rules including a matching to the HMχPT result in the soft pion region.

This can be compared with the LEET results [27] only after the hard gluon effects have been incorporated into these and can thus only been done in the future. There even is a school which assumes that the perturbative contribution is dominant for the $B \to \pi$ transition [28]. It is obvious that the LEET limit deserves further studies which certainly will be done in the near future.

It is also interesting to compare our method and our results to the full QCD LC sum rule calculation. The authors of Ref. [29] in fact considered the heavy quark limit of their full QCD LC sum rule calculation. Their results differ from $L_a(v \cdot p)$ and $L_b(v \cdot p)$ only by a simple transformation which were given in Eqs. (47) and (46) in their paper. There is a subtle difference from our results, though, which can be seen by letting $\chi(1-u)/u \equiv \nu$. It is only after taking $\nu/v \cdot p \to 0$ that the results of the two calculations fully agree. This limit may need further understanding. Numerically the QCD LC sum rules gave stable results for

$q^2 \leq 17$ GeV^2 which is consistent with the HQET LC sum rules requiring $v \cdot p \geq \omega_c/2 \sim 1$ GeV. These two methods could be in principle the same, provided that all the sub-leading corrections have been included in the calculations. Lacking such powerful calculations, the effective theory calculates a physical quantity in a most thorough and least complicated way through clearly separating the perturbative and nonperturbative parts of the quantity. Therefore in certain appropriate regions of the pion's phase space the HQET calculation may give more reliable results. Practically speaking the method is to calculate form factors at the hadronic scale, and the perturbative contribution is accounted for by multiplying in renormalization factors.

It is physically useful to reconstruct the conventional form factors defined in Eq. (1) from the results combining LC-HQET sum rules and HMχPT, by using the relation Eq.(9). In Fig. 12 (a) we present $f_+^{B\to\pi}(q^2)$, which is directly measurable in semi-leptonic decay involving light leptons. In Fig. 12 (b) we present our result for the scalar form factor $f_0^{B\to\pi}(q^2)$. The scalar form factor contributes to the decay $B \to \pi\bar{\tau}\nu_\tau$ and also enters the factorized amplitudes in non-leptonic two-body B decays. For comparison, the results from the lattice QCD simulations by APE [30], UKQCD [31], FNAL [32] and JLQCD [23] are also shown in the figures.

In Table I we compare our results to the results of other model calculations which includes the conventional HQET QCD sum rule calculation of Ref. [33]. We find that our results for $f_+^{B\to\pi}(q^2)$ are about $14 \sim 33\%$ larger than those from the full QCD LC sum rule [1, 34]calculation which also includes α_s corrections.

TABLE I: Comparison between different approaches for the universal functions $L_a(v\cdot p)$ and $L_b(v\cdot p)$, and the form factor $f_+^{B\to\pi}(q^2)$ at some different kinematical points.

	$L_a(v \cdot p)$ [GeV$^{1/2}$]		$L_b(v \cdot p)$ [GeV$^{1/2}$]			$f_+(q^2)$	
$v \cdot p$ [GeV] or q^2 [GeV^2]	1.50	2.64	1.50	2.64	0.0	6.0	12.0
LC-HQET SR (NLO)	——		——		0.42 ± 0.04	0.54	0.77
LC-HQET SR (LO)	0.13	0.06	0.34	0.32	0.36	0.47	0.68
HQET SR in LO [33]	0.15	0.13	0.25	0.21	0.24	0.32	—
LC-QCD SR [1, 34]	——		——		0.28 ± 0.05	0.42	0.66

Finally, in order to be concrete, we give our prediction for the decay width of the decay $B^0 \to \pi^- e^+ \nu_e$ using our form factors. We obtain

$$\Gamma = \left|\frac{V_{ub}}{4.08 \times 10^{-3}}\right|^2 (1.46 \pm 0.30) \times 10^{-16} \text{ GeV}. \tag{78}$$

By taking $|V_{ub}| = (4.08 \pm 1.18) \times 10^{-3}$ from inclusive measurement of $B \to X_u\ell\nu$ [35], we get $\Gamma = (1.46 \pm 0.30^{+0.97}_{-0.72}) \times 10^{-16}$ GeV, where the uncertainties are from that of form factors and $|V_{ub}|$ respectively. On the other hand, from the experimental result given in Ref. [36], $\tau_{B^0} = (1.55 \pm 0.03)$ ps^{-1} and Br$(B^0 \to \pi^- e^+ \nu_e) = (1.8 \pm 0.6) \times 10^{-4}$, we extract $|V_{ub}| = (2.94^{+0.33}_{-0.24} \pm 0.50) \times 10^{-3}$ with the uncertainties from the form factors and the experiments, respectively.

Let us also comment on Ref. [11] which gave leading order results for L_α. In addition to that the analytical expressions of the sum rules differ from ours by a factor of 2, the
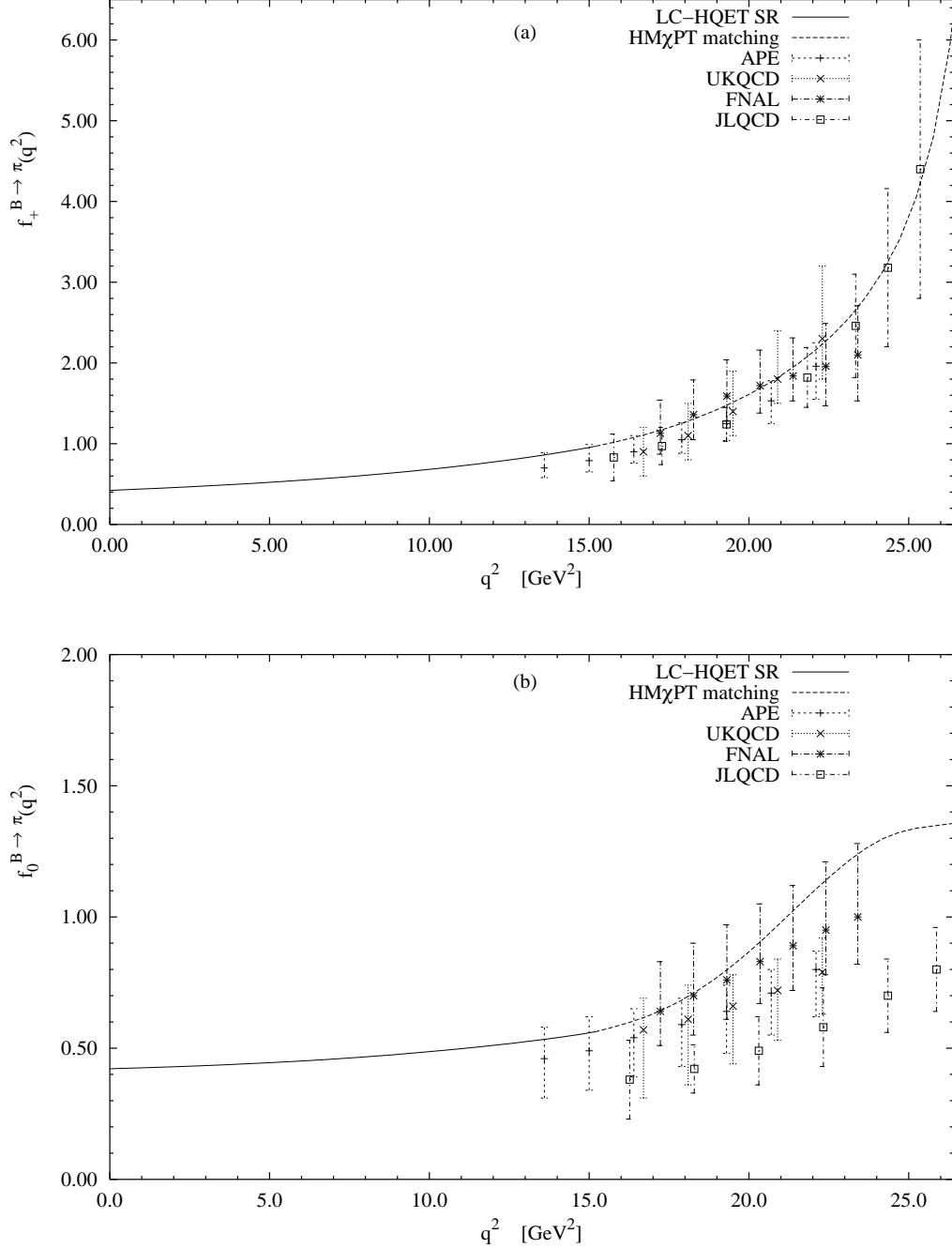
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FIG. 12: The form factors $f_{+,0}^{B\to\pi}(q^2)$ from combining LC-HQET sum rules and HMχPT, and from the lattice QCD simulations by APE [30], UKQCD [31], FNAL [32] and JLQCD [23].

numerical input used in Ref. [11] is quite different, such as the choice of the energy scale μ and the determination of the sum rule window. This naturally affects the final results on $L_\alpha(v \cdot p)$.

To summarize, we have applied the LC sum rule method to calculate the $B \to \pi\ell\nu$ weak decay form factors to order $1/m_Q$ in the framework of HQET. We have calculated the leading and the relevant sub-leading universal form factors. Our form factor results have been matched to the appropiate soft pion results. We have also discussed the large pion

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energy limit of our results. The full QCD LC sum rules and lattice QCD results have been compared with. In the future we are planning to include perturbative QCD corrections, try to incorporate the large energy effective theory into the QCD sum rule technique and perform detailed phenomenological analysis.

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